Mathematical Model for Fuzzy Systems

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*Abstract :*Many Models if not all have a mathematic Paradigms for computations , from these models a fuzzy system .Most of them have a source(s) of information's formal knowledge ,data and various knowledge .Also method(s) of acquisition to mathematical or optimization(learning) or knowledge based ,such these models approach to be as mechanistic ,or fuzzy such differential equations, neural networks, rule-based model which represent a fuzzy model has a mathematical paradigm. This paper is lighting on a mathematical steps in fuzzy logic approaches which is mainly a fuzzy systems (modeling) with it steps .

Key words : Fuzzy system, fuzzy regions, fuzzy rules.

1.Introduction :

Many models have a practical applications ,a fuzzy system is one of them since most application depend on a mathematical framework ,a system description a crisp if input and output data are crisp (or fuzzy) and mathematical framework as functional analysis ,linear algebra (extension principle).While a system describe to be fuzzy system if inputs are crisp or fuzzy but the resulting output data be fuzzy and so since a mathematical framework as fuzzy relation calculus, fuzzy inference .So there is need to understand the model of fuzzy system to be able to separate between it and other systems ,also fuzzy system be static or dynamic not like others .
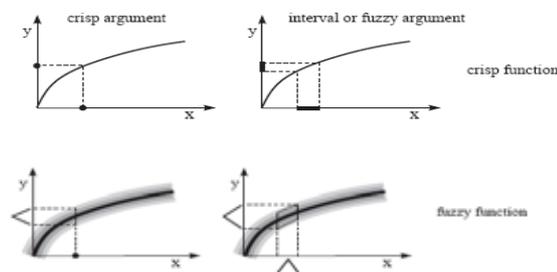
2.Fuzzy system:

Definition1:(Babuˇskay & Verbruggeny & Hellendoorn, Reznik)A static or dynamic system which makes use of fuzzy sets or fuzzy logic and of the corresponding mathematical framework is called *Fuzzy System.*

That is it depends on the variables and its values in fuzzy sets ,and the operations on it(values of variables),which are fuzzy sets within the system ,either to process it as inputs or as outcomes from a system .Through this, fuzzy system use fuzzy set theory to maps inputs to outputs.

There are number of ways fuzzy sets can be involved in a system,such as: (Babuˇska &Verbruggen & Hellendoorn)；

⊕*In the description of the system.* A system can be defined, as a collection of (if-then)rules with fuzzy predicates, or as a fuzzy relation.

✦*In the specification of the system's parameters.* The system can be defined by an algebraic or differential equation, in which the parameters are fuzzy numbers instead of real numbers.

✦*The input, output and state variables of a system may be fuzzy sets;* Fuzzy inputs can be readings from unreliable sensors ("noisy" data), or quantities related to human perception, such as comfort, intelligent, …etc. Fuzzy systems can process such information, while a conventional(crisp) systems could not do .

Figure(1) Implementing fuzzy logic on functions

A fuzzy system can simultaneously have several of the above cases.A fuzzy system can be classified into many kinds depending on :

1. Used space approaches ,since fuzzy systems are apply in numerous purposes(through applications and operations) ,such as; classification(recognition) problems(Robert),control problems(Kobayashi & Torioka),prediction, modeling ,and data analysis(Reznik ,Math),such Fuzzy classifier ,Fuzzy controller ,…,also a fuzzy system can approaches on approximation by Fuzzy systems approximation (Lewis & Burrus).

2. The second base for classification is the number of input variable(s) and output variable(s) .The system which is expressed (the variables)by rules ,such as;

   ✳ *Single Input-Single Output (SISO)fuzzy system .*
   ✳ *Multi Inputs-Multi Outputs (MIMO) fuzzy system .*
   ✳ *Multi Inputs-Single Output (MISO) fuzzy system .*

If a system description as a conventional (crisp) system dealing with crisp input data (such real inputs),with mathematical framework such; functional analysis ,mathematical analysis .linear algebra, topology,...etc ,then the resulting output data traditionally be crisp , While if dealings with a fuzzy inputs data through extension principle then the resulting output data also fuzzy output data .But when a system described as a fuzzy system dealings with a crisp or fuzzy input data through mathematical framework such fuzzy relational calculus or fuzzy inference , then a resulting output data is for both cases as fuzzy output data .This all clarify that a mathematical framework of system determine if it fuzzy or conventional.

The fuzzy system (in classical design) is performed (for all kinds) through three stages(Kobayashi & Torioka, Reznik, Kaarna,Robert).

   1. *Fuzzification.*
   2. *Fuzzy Processing(Fuzzy Inference).*
   3. *Defuzzification.*

For instance ,a fuzzy system for either modeling or control have similar structure (Kobayashi & Torioka, Kaarna).The mechanism of a system ,began with measurements of the outside world ,which are input variables ,in the form of crisp

data transform by fuzzification into linguistic values ,then the linguistic variables values are processed by the fuzzy rules in the rules base ,and design which rules to be applied (it with form if-then)then design the computational unit (inference) which will generally lead to outputs ,outputs expressed in fuzzy sets are transformed by defuzzification into non fuzzy (crisp)outputs as the outputs of the system to the outside world ,again .

As say, fuzzy classifiers are one applications of fuzzy set theory expert knowledge is used and can be expressed in a very natural way using linguistic variables ,which described by fuzzy sets (Robert).Numerical data are also handled as fuzzy patterns ,and some times there is more over variables to each class, also the variables may be features of a class and the linguistic terms corresponding to it a values of that features ,so the known of this features is helpful to recognition a pattern from others .The form of a rules in a fuzzy classifier as ,for instance(Kaarna);

$$\textit{If (feature) is (linguistic term) then class number (name)} \quad \dots \ (1)$$

When a system deals with many variables(features) the patterns space may be the unit square $[0,1]^2$ for two variables ,and higher unit for more variables .

Different methods have been developed for using fuzzy set theory to model system, such as; fuzzy linear regression methods ,fuzzy based on cell structures(Math),knowledge_based method by(Reznik).In this side of research will talking about a rule_based fuzzy models which are from a knowledge_based systems .

2.1 Fuzzification:

The Fuzzification is a first operation in a fuzzy system on inputs data, It is a process mapping a crisp input value of a system into a fuzzy input (to a corresponding degree of membership from 0 to 1by *MF*.) ,that is to associate (linguistic variable(s)) with a fuzzy set(fuzzy numbers) (Yao & Li &Yuan). Since each linguistic variable have values can be described linguistically to a sets of its(variable) values .we can determining number of linguistic terms as we will see for a system .Some times, the inputs are as features of classes ,as in Fuzzy classifier.

Mathematically ,fuzzification can given a definition as:

Definition2:*Fuzzification* is a mapping *F* of the crisp input domain *I* ,with *x* into a set *A* with fuzzified input X (Kaarna).That is as ; $F : I \rightarrow A$

The inputs are mapped into a fuzzy sets (numbers) by drawing a line up from the inputs to the input membership function(s)(determined for a task) and making the intersection point(s).These input membership function(s) can represent linguistic term(s) which a fuzzy set(s) such as large ,small ,old ,young ,…,etc .The membership functions could then represent the amounts of tension ,when choosing the input membership functions ,definition of what we mean by "large" or any others terms may be different for each input.

2.2 Fuzzy Rules and Rule Base:

Fuzzy rule is a form serves to describe ,in linguistic terms(words) a qualitative relationship between two or more variables(Kobayashi & Torioka, Yao & Li &Yuan) that is, it joint the input variable(s) and its(variables) influence with the output variable(s) and these may be linguistic or scalar .

This rules through the relationship is as(if_then) form (Reznik,Kobayashi& Torioka ,Kaarna ,math),with vague predicates(information),and a major mathematical principle that these rules are depend on it ,is the Extension Principle , which operate

on fuzzy sets from inputs to outputs, and so the work of these rules is mainly through this principle   .

   We can write a general form of these rules as(math);

If antecedent part   then consequence part   ...   *(2)*

   Both antecedent part and consequent part has a propositions for values of input variable(s) and output   variable(s),respectively .The antecedent (or called premise) propositions is a fuzzy propositions or crisp (Babuška,Thuillard)of the type (x is A)or (x is [0,1]) in crisp case, where x is a linguistic variable and A is a linguistic term(constant) defined by fuzzy set on the range of universal set U of variable x .This part is represent a condition or (if part) that it must satisfies to get (through a relationship) results(result part) .

   In many approaches for these rules in fuzzy systems ,we have many linguistic variables (such as in control or classification systems),and so each variable have its linguistic term(s),then these inputs (fuzzy terms) can be combined logically by "*AND*","*OR*"(Robert, Garibaldi &John),to perform values for all(or may be most) expressed inputs .So to write this situation as ;

   $x_1 \, is \, A_1 \, OR / AND \, x_2 \, is \, A_2 \, OR / AND....$ ...   *(3)*

for x_1 , x_2 ,... ,are linguistic variables ,and A_1 , A_2 ,... ,are linguistic terms(fuzzy sets) .

   A situation where a variable has many linguistic terms(fuzzy terms) as ;

$x_1 \, is \, A_1 \, OR / AND \, A_2$ .... *(3-85)*

A_1 and A_2 linguistic terms for x .

   The consequent part(result part) is similar to the form of antecedent part as output linguistic variable(s) and its linguistic terms (values) .This part (or called then part) is an action to be performed in the output(s) ,it may be a fuzzy proposition or crisp(or single value ,real value) (Kaarna, Reznik).

   In many approaches of fuzzy systems ,the consequent part consists of more than one output(singleton) or linguistic variable(s) and these variables (similar situation to antecedent part) combined logically by operators "*AND* "or "*OR* " but in very limited situations   .

   Depending on the form of the consequent part of fuzzy rules two main types(of rule_based fuzzy models)are distinguished (Kobayashi & Torioka, Reznik, Kaarna)

   ◈ *Linguistic fuzzy model* .
   ◈ *Takagi_Sugeno fuzzy model* .

   Here upon we can write the general form of fuzzy rules as(Math,Kaarna,Kobayashi & Torioka) ;

   $x \, is \, A \, then \, y \, is \, B$ ...*(4)*

where x is an input linguistic variable , y an output linguistic variable, and A , B are linguistic terms for input and output ,respectively .

   Or in more generalization for many variables ;

   $if \, x_1 \, is \, A_1 \, OR / AND \, x_2 \, is \, A_2 \, OR / AND ... OR / AND \, x_n \, is \, A_n$
                                                           ... *(5)*
   $then \, y \, is \, B$

where $x_1, x_2,..., x_n$ are input linguistic variables , and $A_1, A_2,... , A_n$, B are linguistic terms for input and output ,respectively .

   And a form for multi_input and multi_output ,and their special cases will formed similarly .Then the form case in which many output variables as;

if x_1 *is* A_1 *OR / AND* x_2 *is* A_2 *OR / AND ...OR / AND* x_n *is* A_n

then y_1 *is* B_1 *AND* y_2 *is* B_2 *AND....AND* y_m *is* B_m

$$\qquad \ldots \quad (6)$$

where $B_1, B_2,..., B_m$ are linguistic terms for output , m is number of output variables .

In pattern recognition fuzzy systems (which are a special case of singleton).The output(consequent part) will be the number or (name) of class as follows (Robert,Kaarna) ;

Example.1: The linguistic variables are a features of class, its linguistic terms are {low, medium ,high}.The number of classes is *3* classes ,and these classes are{circle , cube ,triangle} ,the rule in a recognition system as ;

if feature A is low AND feature B is medium

then class $= 2(or\ circle)$

$$\qquad \ldots \quad (7)$$

as example .

In a simplified version of fuzzy if_then rules ,a real number is used in consequent part(instead of the fuzzy number B ,or a linguistic term) (Reznik,Babuška,Kaarna)in equation(2) ,as example to say ,which can be written as follows ;

$$R_1 : if \quad x \quad is \quad A_i \quad then \quad y = b_i \qquad \ldots \quad (8)$$

$i = 1,..., k$,for single input variable .

Or as;

$$R_i : if \quad x_1 \ is \ A_{i1} \quad AND \quad x_2 \quad is \ A_{i2} \quad AND \quad .... AND \quad x_n \quad is \quad A_{in}$$

$$then \quad y = b \qquad \ldots \quad (9)$$

for multi_input variables,where b_i is real number, n is number of input linguistic variables, i is number of linguistic terms for each variable .

We can write the linguistic variable as vector $x = (x_1,..., x_n)$,that is n_dimensional input vector ,this type used also in pattern recognition .

Recently these fuzzy rules have frequently been used ,because the simplicity of the fuzzy reasoning (Kaarna).This case as called "singleton model " as it occurred in (Reznik).

Here by ,we become know that fuzzy rules mapping the input variable(s) and reflect its influence into the output variable(s) .Some time no influence (Kobayashi & Torioka) ,(this case noted in control systems),or that is mean nothing in process of system so it will negated .It is just use the influenced rules ,even if not use all fuzzy(linguistic term) values of a linguistic variable(s) (input, output) .

In fuzzy rules needed two different (or more) MFs ,for input and for output ,in some rules needed only one MF to define input and reflect the influence of input(s) by some MF on output(s) (output(s) define by same MF with input(s)) ,so need just change the membership degree from input to output, (and not the domain of output) .

A fuzzy system consists of a set of (linguistic) rules (not one rule), called a rule base (rule table) (Reznik ,Kobayashi & Torioka,Kaarna),this set can written as ;

$$R = \{R_i : i = 1,2,..., k\} \qquad \ldots \quad (10)$$

i is number of fuzzy rules in system .

This rules characterize the behavior of a system through a linguistic information contained in linguistic variables and linguistic terms ,such as human experts(Kaarna),from this table produce the response(process) of fuzzy system on the input variable(s) (and its values in fuzzy set) through ,the result of each fuzzy rule in rule base(table) ,that is the fuzzy output of each rule .The final result is the union of all the rules results(Kobayashi ,Torioka),as follows;

$$R_1 \cup R_2 \cup .... \cup R_k = \bigcup_{i=1}^{k} R_i \qquad \qquad ... \quad (11)$$

where R_i is the result of applying the $i\,th$ rule (output fuzzy region), and k is a number of rules applied on the input variable(s),the number of rules is restricted by all fuzzy values of input and output variables ,through combine each value for input variable(s)(linguistic term) with (at less) one fuzzy value(s) of output variable(s) (this similar to Cartesian product of traditional sets) .Depending on the system, it may not be necessary to evaluate every possible input combination, since some many rarely or never occur(Robert).By making this type of evaluation ,usually done by experienced operator(try_experience) (Kobayashi & Torioka), fewer rules can be evaluated ,then thus simplifying the process logic and perhaps even improving the fuzzy logic systems performance(this usual exists in control systems)(Robert),and this what we mean by say we attended in influence rules in system .

Note: Some times we have no value for one or more of (input ,output) variables .In this case ,a linguistic value mean all range ,or we deals with numerical data (real numbers, which is a special subset hood of fuzzy numbers),and the MF as;

$$\mu_A(x) = \begin{cases} 1 & ,if \quad x = a \\ 0 & ,o.w \end{cases} \qquad ... \quad (12)$$

Where a is real number .

*Example3.41:*We have two linguistic variables x_1 , x_2 ,the linguistic values for x_1 are{small ,medium ,large},and for x_2 are {short ,long }.The rules which extracted here , such;

$$if \quad x_1 \quad is \quad large \ then \quad class = 3 \ (or \ y \ is\,large) \qquad ... \quad (13)$$

but we have two variables ,then we can write it as

$$if \quad x_1 \quad is \quad large \ AND \quad x_2 \ in[0,1] \ then \ class = 3 \ ) \qquad ... \quad (14)$$

or as ordered pair ;

$$(large ,[0,1]) \rightarrow class = 3 \qquad ... \quad (15)$$

The range [0,1] contain all possible values for x_2 .

Note: Crisp logical rules may be highly accurate in classification problems because it is easy to recognize when over fitting occurs .It may be so simple with the fuzzy rules [20] .Fuzzy rules carry more information but their interpretation is more difficult .

2.3 Defuzzification :

The defuzzification procedure is a final stage of fuzzy operations ,it is a needed to maps a fuzzy output of a fuzzy system to a crisp output of the system .

Mathematically can define as ;

Definition3.27:Defuzzification is a mapping D of the set $J(Y)$ with fuzzified outputs Y ,into the crisp output domain O with Y (Kaarna).

$$D : J(Y) \rightarrow O$$

In many applications instances, it is desired to come up with a (single)crisp output vector form .That is output fuzzy set(s) must be defuzzified ,that is mean defuzzify the outcomes (through the system) for inputs that were fuzzified previously in starting system operation ,So it is invert the work of fuzzification operation .From these applications a real control system(Kobayashi & Torioka),controller output should be used to control a real object or process ,so it be needed to know a crisp value(in a real world)for every output signal(fuzzy).A defuzzification produces this value on the basis of output MFs .

Note that ,the output(s) and input(s) ,sometimes for a fuzzy system , are crisp but in spite this ,similarity fuzzy control gives a rather simple to use method ,for producing high quality controllers complicated input(s) or output(s) characteristics .

Also in fuzzy systems for other applications of purposes its aim to classify or recognize(identify),the objects with vague information on it ,so output has no wide change in outcomes(Kobayashi & Torioka, Robert).For instance if one was trying to classify a letter drawn by hand to a drawn by tablet ,in ultimately the FS ,would have to come up with a crisp number to tell the computer which letter was drawn, this number which crisp is obtained in a defuzzification process also .

The TS fuzzy system do not needed this process ,due to the nature of consequent part of rules in the system(Kobayashi & Torioka, Reznik,Math).But in Mamdani (Linguistic fuzzy) system on output distribution is only needed if a crisp output(class) is needed .

The defuzzification process is(in spite its importance) not very difficult, and does not significantly influence on the system performance. So there are a few methods have been developed (Yao & Li &Yuan ,Kobayashi & Torioka),and to be helpful to solve the chosen problem .From a very brief of the most widely used defuzzification procedures (methods) and schemes that have been proposed (Kobayashi & Torioka, Kaarna,Yao & Li &Yuan);

- o *Center of area (gravity) defuzzification CoG[1] .*
- o *Center of (maxima) Largest area defuzzification CLA .*
- o *Mean of maxima defuzzification .*
- o *First of maxima defuzzification (FoM).*
- o *Middle of maxima defuzzification (MoM).*
- o *Height defuzzification .*

In spite of these methods are so many they are all be sufficient and we can apply any one of them ,the choice of the defuzzification procedure is based mainly on personal performance(Kobayashi & Torioka),with the Mamdani inference model ,the center of gravity (CoG) defuzzification in usually (Reznik).

The first two methods tend to produce an integral output considering all the elements of the resulting fuzzy set with the corresponding weights. Other methods take into account just the elements corresponding to the maximum points of resulting MFs (so this joint with their called)(Kobayashi & Torioka).

There are some depended criterias for choice a defuzzification method that suitable ,also it is as feature can commend for methods;

1) *Continuity :*which means that a small change I the input of a fuzzy system(usually in controller systems) should not result in a large change in output of a system .

2) *Disambiguity*: which means that a defuzzification method should work in any situation ,this is not always the case. The CLA defuzzification method cannot make choice when we have two equal areas .

3) *Plausibility*: which means that a defuzzification output lies approximately in the middle of the support of the resulting MF, and has a high degree of membership ,the center of area method does not satisfy this property ,although center of area output lies in the middle of support set ,its membership degree may be one of lowest possible .

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[1] Some times as its name center of area denoted as (CoA) .

4) *Computational complexity criterion:* is particularly important in practical applications of fuzzy systems .The methods that dealing with a maximal point are fast methods while center of area method is slower.

It is some times depends on the shape of output MFs,whether max_min composition _based inference ,the middle of maxima method is a faster .

2.3.1 Center of Gravity (Area of Mass)

Different authors called it with different names ;as the center of area as in (Kobayashi &Torioka)or center of gravity in (Reznik)or as center of mass .

This method is the most well_known defuzzification method(Kobayashi & Torioka),it is work as take center of area (gravity)[2] of output fuzzy set that find a center of area to come up with one of (or more)crisp number ,and it is work on two cases for types of the output signal with corresponding membership degrees ,that is when have a finite fuzzy set .Through the form of fuzzy set thus must be discretized to be able to compute the center of area;

$$y' = cog(B)' = \frac{\sum_{i=1}^{k} y_i \mu_{B'}(y_i)}{\sum_{i=1}^{k} \mu_{B'}(y_i)} \qquad \dots \quad (16)$$

where y' is the center of area ,

$\mu(.)$ is the membership degree(in class) at value y_i ,

and k is number of elements y_i in Y .

The output vector y' is first normalized (Yao & Li &Yuan,Kobayashi &Torioka)to capture on a values to be within possible ,in each one from these situations discrete or continuous .

In the continuous case we obtain (Kobayashi &Torioka);

$$y' = \frac{\int_S y\,\mu(y)\,dy}{\int_S \mu(y)\,dy} \qquad \dots \quad (17)$$

where $\int$ is the classical integral ,and S is fuzzy set ,$\mu(.)$ is a combined membership function .

This method determines the center of area below the combined MF ,since the integral compute the area under curve of MF by a determined region (fuzzy set) S .

Note: It can be seen that this defuzzification method takes into account the area of S as the whole area (Kobayashi & Torioka).Thus ,if the areas of two fuzzy sets are overlapped ,then the overlapping area is not be problem(and not paid any particular attention) . By integration ,which is computationally rather complex and since it work on a region , therefore resulting in quite slow inference .

2.3.2 Center of Largest Area:

The center of largest area CLA is used in case when the area S is non convex ,i.e. consists of at least two convex fuzzy(sub)sets ,which are not overlapped;

$$\forall\, x, y \in S \Rightarrow \exists\, \lambda \ni 0 \geq \lambda \geq 1 \;,\; \lambda x + (1-\lambda) y \notin S \qquad \dots \quad (18)$$

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[2] Note different authors some times named the same procedure differently .

then method determines ,convex fuzzy subset with the largest area of this particular fuzzy subset(one fuzzy set from the fuzzy sets in the area S) .

It is difficult to represent this defuzzification method formally because it involves ,firstly finding the convex fuzzy (sub)set and then computing their areas ,also the defuzzification result is biased towards a side of one MF .

2.3.4 First of Maxima (Last of Maxima)

First of maxima FoM use S (area) and take a smallest value of the domain S ,with maximal membership degree in S ,through a three steps(stages) .

Let $$hgt(S) = \sup_{x \in S} \mu_S(x) \qquad \qquad \dots \quad (19)$$

be the highest membership degree of S ,

and let $\{x \in S : \mu_S(x)\} = hgt(S)\}$ be the set of domain elements with degree of membership equal to $hgt(S)$, then y is given by ;

$$y = \inf_{x \in S} \{x \in S : \mu_S(x) = hgt(x)\} \qquad \dots \quad (20)$$

The alternative version of this method ,is called Last of Maxima, and is given as;

$$y = \sup_{x \in S} \{x \in S : \mu_S(x) = hgt(x)\} \qquad \dots \quad (21)$$

2.3.5 Middle of Maxima

Middle of maxima MoM is very similar to first of maxima or last of maxima instead of determining y to be the first or last from all values when S has the maximal membership degree .This method takes the average of these two values formally it given as;

$$y = \frac{\inf_{x \in S} \{x \in S : \mu_S(x) = hgt(x)\} + \sup_{x \in S} \{x \in S : \mu_S(x) = hgt(x)\}}{2} \qquad \dots \quad (22)$$

2.3.6 Mean of Maxima

This method does not differentiate between elements of the combined MF ,instead it considers all of them and takes the average .If the MF has n maximal points ,then the output will be ;

$$y = \frac{\sum_{i=1}^{n} \{x \in S : \mu_S(x) = hgt(x)\}}{n} \qquad \dots \quad (3\text{-}105)$$

x is appoint at which the MF is maxima , n is number of times the output distribution reach the maximum .This technique takes the output distribution and find its mean of maxima to come up with one crisp number.

2.3.7 Height Defuzzification :

Height is a method takes the peak value of each MF .Thus nether support nor shape of MF play a role in the computation of y (Kobayashi &Torioka) .

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